For parametric surface matching / identification, it is often very helpful to think about the grid curves i.e. holding one parameter constant.

e.g. 13 $\langle u\cos v, u\sin v, v\rangle$

u const $\longrightarrow$ helixes

v const $\longrightarrow$ lines (emanating from z-axis parallel to xy-plane)

14 Both families of grid curves are parabolas.

Remember that a surface of the form

$$z = f(x,y) \qquad \text{(i.e. a \underline{graph})}$$

has a natural parametrization:

$$x = x$$
$$y = y$$
$$z = f(x,y)$$

Q: Can you parametrize the surface

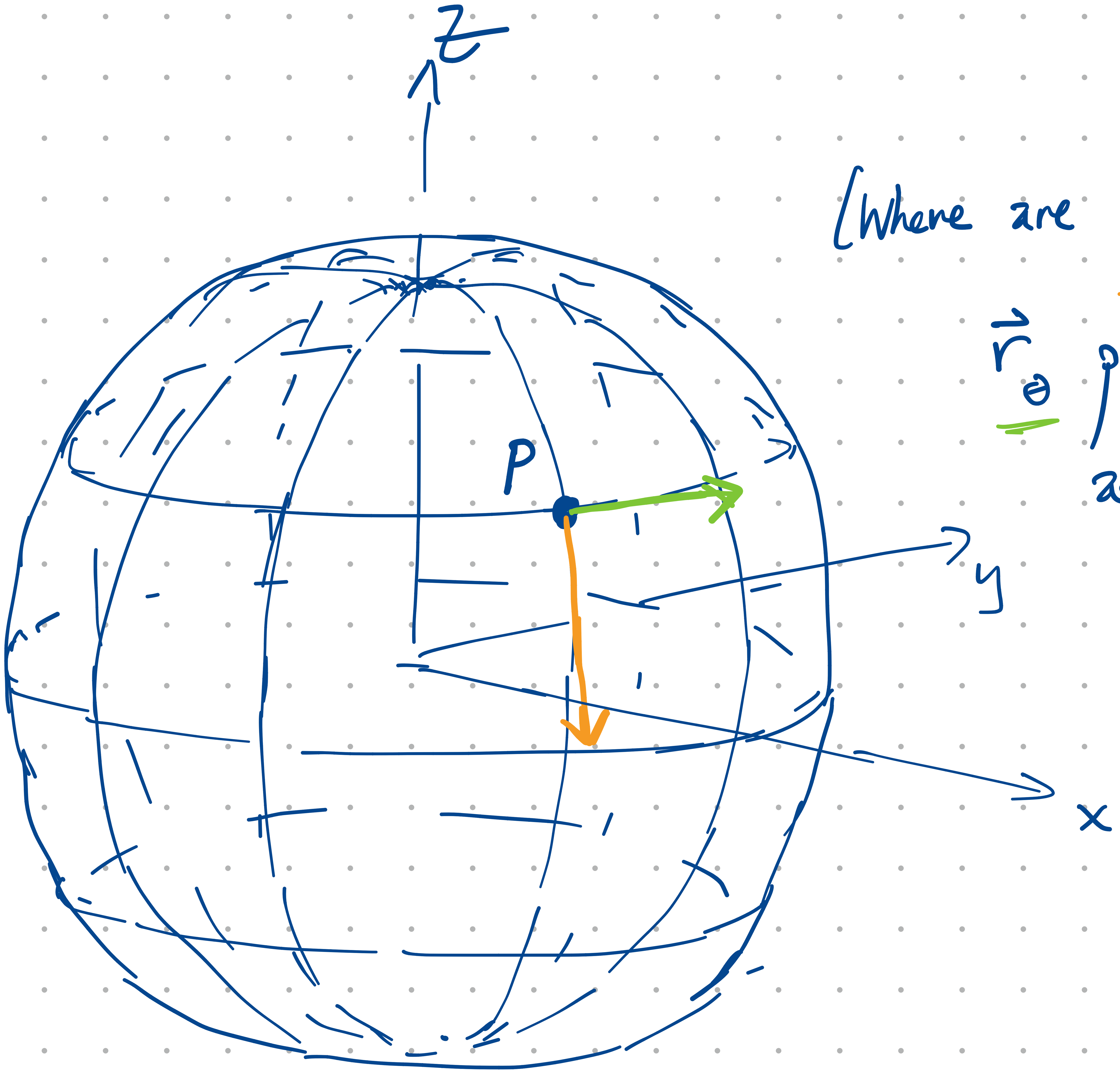
$$x^2+y^2+z^2=4 \ ?$$

$$x = 2\sin\phi\cos\theta$$
$$y = 2\sin\phi\sin\theta$$
$$z = 2\cos\phi$$

$0 \le \phi \le \pi$

$0 \le \theta \le 2\pi$

Can you write down the outwards pointing unit normal vector to this surface (in terms of ϕ, θ) ?



(Where are $\vec{r}_\phi$ and $\vec{r}_\theta$ pointing, at the point P?)

Q: How to decide whether to take

$$\vec{r}_\phi \times \vec{r}_\theta \quad \text{or} \quad \vec{r}_\theta \times \vec{r}_\phi \ ?$$

A1: Interpret the two derivatives geometrically and then apply RHR.

A2: Pick one of the two. Compute it. Then plug in an "easy" point, and see if your cross product points in the correct direction there.

If yes ✓

If not, just take its negative.

Final ans to q:

$$\frac{\vec{r}_\phi \times \vec{r}_\theta}{|\vec{r}_\phi \times \vec{r}_\theta|} \quad (*)$$

In this particular problem, we already have a Cartesian eq:

$$x^2+y^2+z^2=4$$

so a normal vec is given by $\langle 2x, 2y, 2z\rangle$

This is outwards, as desired.

$$\frac{\langle 2x, 2y, 2z\rangle}{\sqrt{4x^2+4y^2+4z^2}} = \frac{\langle 2x, 2y, 2z\rangle}{4}$$

$$= \frac{1}{2}\langle x, y, z\rangle$$

$$= \langle \sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi\rangle.$$

(this is the same as (*), as you can check)

Please refer to 213 notes for answers to poller survey.